On the Connectivity and Multihop Delay of Ad Hoc Cognitive Radio Networks

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Poisson Primary + Poisson Secondary
Primary Network:
- Active primary Txs form a 2-D Poisson process.
- Primary Rxs are uniformly distributed within Tx range of their transmitters.
- Slotted transmission.
For two randomly chosen secondary users:

- can they communicate via multihop relay with finite delay (connectivity)?
- how does multihop delay scale with S-D distance?
**Multihop Transmission in Secondary Network**

**Unique Challenges:** interaction between primary and secondary networks

- Existence of a link depends on Tx/Rx activities of nearby PUs.
- Delay at each hop = propagation delay + waiting time for an opportunity.
Topological Links in Secondary Network

Topological Links: formed by secondary users within each other’s Tx range.
Communication Links in Secondary Network

Topological Links: formed by secondary users within each other’s Tx range.

Communication Links: topological links that see bidirectional opportunities.
Connectivity of the Secondary Network

Connectivity:
- defined by finiteness of min multihop delay btw. two randomly chosen SUs.
- depends on the density of SUs and the traffic load of PUs.
Connectivity: Static Case

When PTxs/PRxs are static over time:

\[ \lambda_{PT} \text{ (Density of PTxs)} \]

\[ \lambda_S \text{ (Density of SUs)} \]

Disconnected

(Instantaneously) Connected

Connectivity $\overset{\Delta}{=} \text{Finite MMD} = \text{Existence of inf. component connected by comm. links.}$
Connectivity Region

- \( \forall (\lambda_S, \lambda_{PT}) \in \mathcal{C} \), there exists a \textit{unique} infinite connected component.
- \( \lambda_{PT}^*(\lambda_S) \) monotonically increases with \( \lambda_S \).
- The critical density of secondary users: \( \lambda_S^* = \lambda_c(r_{tx}) \) (CD of homogenous networks).
- The critical density of primary Tx: \( \overline{\lambda_{PT}^*} \leq \min \left\{ \frac{1}{4(R_t^2-r_{p}^2/4)}\lambda_c(1), \frac{1}{4(r_t^2-r_{p}^2/4)}\lambda_c(1) \right\} \).
Connectivity: Dynamic Case

When temporal dynamics of PTxs/PRxs are sufficiently rich:

- Connectivity = Existence of inf. component connected by topological links.
- Connectivity of SUs is independent of the primary traffic load.
MMD under Negligible Propagation Delay

When instantaneously connected:

- MMD is asymptotically independent of the S-D distance:

  \[
  \lim_{d(\mu, \nu) \to \infty} \frac{t(\mu, \nu)}{g(d(\mu, \nu))} = 0 \text{ a.s.,}
  \]

  where \( g(d) \) is any monotonically increasing function of \( d \) with \( \lim_{d \to \infty} g(d) = \infty \).

When intermittently connected:

- MMD grows linearly with the S-D distance:

  \[
  \lim_{d(\mu, \nu) \to \infty} \frac{t(\mu, \nu)}{d(\mu, \nu)} = \beta \text{ a.s.,}
  \]

  where the value of \( \beta > 0 \) depends on \( (\lambda_S, \lambda_{PT}) \) and the temporal dynamics of interference.
MMD under Negligible Propagation Delay

When instantaneously connected:

When intermittently connected:
With propagation delay:

- MMD scales linearly with S-D distance.
- Scaling rate for an instantaneously connected network can be orders of magnitude smaller than that for an intermittently connected network.

(a) instantaneously connected  
(b) intermittently connected
Conclusion and Future Directions

Static Case:

\[
\lambda_{PT} \quad \lambda_S = \lambda_c
\]

Disconnected

Connected

Dynamic Case:

\[
\lambda_{PT} \quad \lambda_S = \lambda_c
\]

Disconnected

Intermittently Connected

Instantaneously Connected

Future Directions:

- Fading dealt with by a random connection model for continuum percolation.
- Contention among secondary users and interference aggregation.